NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: BACHELOR OF SCIENCE; BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS	
QUALIFICATION CODE: 07BSOC; 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: JANUARY 2020	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SECOND OPPORTUNITY/ SUPPLEMENTARY EXAMINATION QUESTION PAPER	
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MODERATOR:	MR B. OBABUEKI

INSTRUCTIONS
1. Examination conditions apply at all times. **NO** books, notes, or phones are allowed.
2. Answer **ALL** the questions and number your answers clearly and correctly.
3. Marks will not be awarded for answers obtained without showing the necessary steps leading to them (the answers).
4. Write clearly and neatly.
5. All written work must be done in dark blue or black ink.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1. [34 MARKS]

1.1 Determine whether each of the following mappings T is linear, or not. Justify your answer.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^3$, where $T(x, y) = (3y, 2x, -y)$. [5]

(b) $T : P_1 \to \mathbb{R}^2$, where $T[p(x)] = [p(0), p(1)]$. [5]

(c) $T : \mathbb{R}^3 \to \mathbb{R}^2$, where $T(x, y, z) = (x + 1, y + z)$. [5]

1.2 Define the following terms as they are used in linear algebra:

(a) The kernel of a linear mapping. [2]

(b) A singular mapping. [2]

(c) A one-to-one mapping. [2]

1.3 Let V be the subspace of $C[0, 2\pi]$ spanned by the vectors $1, \sin x, \cos x$, and let $T : V \to \mathbb{R}^3$ be the *evaluation transformation* on V at the sequence points $0, \pi, 2\pi$. Find

(a) $T(1 + \sin x + \cos x)$. [2]

(b) $ker(T)$. [5]

1.4 Let F and G be the linear operators on $\mathbb{R}^2$ defined by

$$F(x, y) = (x + y, 0) \quad \text{and} \quad G(x, y) = (-y, x).$$

Find formulas defining the following linear operators:

(a) $3F - 2G$. [2]

(b) $F \circ G$. [2]

(c) G^2. [2]

QUESTION 2. [28 MARKS]

2.1 Let $T : P_2 \to P_2$ be a linear operator defined by

$$T(a_0 + a_1 x + a_2 x^2) = a_0 + a_1(3x - 5) + a_2(3x - 5)^2,$$

and the basis $S = \{1, x, x^2\}$ for P_2.

(a) Find the matrix representation of T relative to S, and denote it by $[T]_S$. [7]

(b) By observing that S is the standard basis for P_2, or otherwise, find the coordinate vector for $\mathbf{p} = 1 + 2x + 3x^2$ relative to the basis S, and denote it by $[p]_S$. [2]

(c) Use the transition matrix you obtained in part (a) above and the result in (b) to compute $[T(p)]_S$. [4]

(d) Hence, determine $T(p) = T(1 + 2x + 3x^2)$, again by noting that S is the standard basis for P_2. [2]

2.2 Consider the bases

$$S_1 = \{p_1, p_2\} = \{6 + 3x,\ 10 + 2x\} \text{ and } S_2 = \{q_1, q_2\} = \{2,\ 3 + 2x\}$$

for P_1, the vector space of polynomials of degree ≤ 1.

(a) Find the transition matrix from S_1 to S_2 and denote it by $P_{S_1 \to S_2}$. [7]

(b) Compute the coordinate vector $[p]_{S_1}$, where $p = -4 + x$, and use the transition matrix you obtained in part (a) above to compute $[p]_{S_2}$. [6]

QUESTION 3. [20 MARKS]

3.1 Prove that the characteristic polynomial of a 2×2 matrix A can be expressed as

$$\lambda^2 - tr(A)\lambda + det(A).$$

[4]

3.2 Suppose $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and $P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) Confirm that P diagonalises A, by finding P^{-1} and computing $P^{-1}AP = D$. [9]

(b) Hence, find A^{13}. [7]

QUESTION 4. [18 MARKS]

4.1 Let $\mathbf{x}^T A\mathbf{x}$ be a quadratic form in the variables $x_1, x_2, \cdots, x_n$, and define $T : \mathbb{R}^n \to \mathbb{R}$ by $T(\mathbf{x}) = \mathbf{x}^T A\mathbf{x}$. Show that $T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x}) + 2\mathbf{x}^T A\mathbf{y} + T(\mathbf{y})$ and $T(c\mathbf{x}) = c^2 T(\mathbf{x})$, for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and $c \in \mathbb{R}$. [8]

4.2 Find an orthogonal change of variables that eliminates the cross product terms in the quadratic form

$$Q(\mathbf{x}) = x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3$$

and express Q in terms of the new variables. [10]

END OF QUESTION PAPER